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Thermodynamics of R-charged black holes in AdS_5 from effective strings

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Abstract: It is well known that the thermodynamics of certain near-extremal black holes in asymptotically flat space can be lifted to an effective string description created from the intersection of D-branes. In this paper we present evidence that the semiclassical thermodynamics of near-extremal R-charged black holes in $AdS_5 \times S^5$ is described in a similar manner by effective strings created from the intersection of giant gravitons on the S^5. We also present a free fermion description of the supersymmetric limit of the one-charge black hole, and we give a crude catalog of the microstates of the two and three-charge black holes in terms of operators in the dual conformal field theory.

Keywords: Black Holes in String Theory, AdS-CFT and dS-CFT Correspondence.

Contents

1. Introduction

Little exact information is available for black holes in AdS_p with $p > 3$. For example, there is a 3/4 mismatch between the free energy of an uncharged black hole in AdS_5 and the free-field calculation of free energy in the dual gauge theory [1]. This mismatch is thought to be due to the strong interactions that are present in the gauge theory in the regime where supergravity calculations are reliable. Calculations both in supergravity [2] and field theory [3] support the view that $f(g_{\rm YM}^2 N_c) \equiv -F/N_c^2 V T^4$ smoothly interpolates between its free field value at $g_{\rm YM}^2 N = 0$ and 3/4 this value at $g_{\rm YM}^2 N = \infty$.

Novel results on the AdS/CFT correspondence [4]–[6] have been obtained by studying states with some large quantum number, like R-charge [7] or angular momentum [8]. Many of these results have yielded new exact comparisons between string theory and a dual gauge theory. In light of all this progress, it is tempting to reconsider black holes in AdS_5 with one, two, or three large charges. On the supergravity side, these black holes are solutions of $d = 5$, $\mathcal{N} = 8$ gauged supergravity. The three charges correspond to the $\mathrm{U}(1)^3$ Cartan subalgebra of the gauge group SO(6). In $AdS_5 \times S^5$, these backgrounds can be described as black holes with three independent angular momenta in the S^5 directions. In asymptotically flat space, a limit of these solutions describes D3-branes with angular momenta in the three independent planes orthogonal to the brane world-volumes. In the dual gauge theory, these

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charged black holes correspond to a thermal bath, uniform on the S^3 spatial slice of the boundary of AdS_5, and carrying three R-charges: the R-symmetry group, of course, is the same SO(6) that describes rotations on S^5.

These R-charged black holes have been studied extensively: see for example [9]–[13]. They have some similarities to the three-charge black holes in asymptotically flat space which figured so prominently in the string theory counting of black hole microstates [14]–[16]. The purpose of this paper is to examine the extent to which similar methods, based on intersecting D-branes, can be used to account for the semiclassical thermodynamics of the R-charged black holes in AdS_5. Our main result is that, under certain plausible assumptions, in the regime where at least one charge is small compared to the others, intersecting giant gravitons do indeed give a microstate counting that agrees with the semiclassical results. It is interesting to note that for the three-charge case, even at zero temperature, a horizon breaks supersymmetry: this is in contrast to the asymptotically flat space case, where BPS configurations exist with a macroscopic horizon.

The organization of the rest of the paper is as follows. In section 2 we describe the R-charged black holes as the classical backgrounds in $d = 5$, $\mathcal{N} = 8$ supergravity. After establishing the relative sizes of the CFT charges to N_c in section 4, in section 5 we show that the semiclassical thermodynamics of the black holes is reproduced (with the aforementioned caveats) by effective strings created by the intersection of distributions of giant gravitons. In section 6 we exhibit a free fermion description of the supersymmetric limit of the one-charge black hole. We conclude in section 7 with some further observations about these black holes and their field theory duals.

The results in this paper are based in part on the senior thesis of JJH [17].

2. R-charged black holes in AdS_5

In $d = 5$, $\mathcal{N} = 8$ gauged type IIB supergravity, the metric for an R-charged black hole in AdS_5 is given by [9]:

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -H^{-2/3} f dt^2 + H^{1/3}\left(f^{-1}dr^2 + r^2 d\Omega_{3,k}^2\right) \tag{2.1}$$

where $d\Omega_{3,k}^2$ is the metric on an S^3, and

$$H_i = 1 + \frac{q_i}{r^2} \tag{2.2}$$

$$H = \prod_{i=1}^{3}\left(1 + \frac{q_i}{r^2}\right) \tag{2.3}$$

$$f = k - \frac{\mu}{r^2} + \frac{r^2}{L^2}H\,. \tag{2.4}$$

Here L is the radius of AdS_5, and k controls the size of the S^3. The value $k = 1$ corresponds to an S^3 of radius 1, and the value $k = 0$ corresponds to the conformal completion of $\mathbb{R}^3$. In all that follows, we assume $k = 1$. The charges q_i are related to the three independent angular momenta in $AdS_5 \times S^5$. The parameter $\mu \geq 0$ describes departures from extremality: at $\mu = 0$ supersymmetry is restored. The mass of the black hole is given by the formula:

$$M = \frac{\pi}{4G_5}\left(\frac{3}{2}\mu + q_1 + q_2 + q_3\right), \tag{2.5}$$

where G_5 is the five dimensional Newton constant. The q_i are not conserved charges, but they are simply related to conserved charges $\tilde{q}_i$:

$$\tilde{q}_i = \sqrt{q_i(\mu + q_i)}\,. \tag{2.6}$$

The scalar potentials for the three U(1) gauge fields are

$$A_t^i = \frac{\tilde{q}_i}{r^2 + q_i}, \tag{2.7}$$

and there are also three real scalar fields (parametrizing directions in the $E_{7,7}/\,\mathrm{USp}(8)$ coset that describes the 78 scalar fields in $d = 5$, $\mathcal{N} = 8$ gauged supergravity) [9]

$$X^i = H_i^{-1}\left(\prod_{i=1}^{3} H_i\right)^{1/3}. \tag{2.8}$$

In general, we will work close enough to extremality ($\mu = 0$) that the difference between q_i and $\tilde{q}_i$ is negligible. Note however that for at least two q_i non-zero, one cannot take μ arbitrarily small without losing the horizon altogether: the radial location of the outer horizon r_+ is the largest double zero of g_{tt}, which leads to

$$f(r_+) = 1 - \frac{\mu}{r_+^2} + \frac{r_+^2}{L^2}\left(1 + \frac{q_1}{r_+^2}\right)\left(1 + \frac{q_2}{r_+^2}\right)\left(1 + \frac{q_3}{r_+^2}\right) = 0\,. \tag{2.9}$$

Let us refer to the smallest value of μ that leads to a horizon as μ_{crit}. For all $\mu > \mu_{\mathrm{crit}}$, there is a regular horizon with finite temperature. For all $\mu < \mu_{\mathrm{crit}}$, there is a naked singularity at $r = 0$. For $\mu = 0$ (the supersymmetric case) the configurations (2.1) have been referred to as "superstars."

For $\mu = \mu_{\mathrm{crit}}$, $f(r)$ has a double zero at $r = r_+$: thus $f(r_+) = f'(r_+) = 0$. These two conditions can be expressed more simply as

$$\begin{aligned} L^2 + q_1 + q_2 + q_3 - \frac{q_1 q_2 q_3}{r_+^4} + 2r_+^2 &= 0 \\ \mu_{\mathrm{crit}} L^2 &= -r_+^4 + q_1 q_2 + q_1 q_3 + q_2 q_3 + \frac{2 q_1 q_2 q_3}{r_+^2}\,. \end{aligned} \tag{2.10}$$

The first of these may be solved explicitly to give r_+/L in terms of q_i/L^2, and then the second may be used to express μ/L^2 in terms of q_i/L^2. The resulting expressions are not simple, so we will not record them here. Let us however quote some special limits.

- $q_2 = q_3 = 0$. In this case, when $\mu = 0$, there is a naked singularity in the five-dimensional solution at $r = 0$, and for $\mu > 0$, there is a horizon at

  $$r_{+(1)}^2 = \frac{\mu L^2}{L^2 + q_1} + O(\mu^2)\,. \tag{2.11}$$

  When the black hole is arbitrarily close to criticality, the outer horizon has vanishingly small size.

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- $q_3 = 0$. When $\mu < \mu_{\rm crit} = q_1 q_2/L^2$ there is a naked singularity at $r = 0$. For $\mu \geq \mu_{\rm crit}$, there is a horizon at

$$\begin{aligned} r^2_{+(2)} &= \frac{L^2+q_1+q_2}{2}\left(-1+\sqrt{1+4L^2\frac{\mu-\mu_{crit}}{(L^2+q_1+q_2)^2}}\right) \\ &= L^4\frac{\mu-\mu_{\rm crit}}{L^2+q_1+q_2}+O[(\mu-\mu_{\rm crit})^2]\,, \end{aligned} \tag{2.12}$$

which is vanishingly small as the black hole approaches criticality.

- $0 < q_i \ll L^2$. There is a naked singularity at $r = 0$ when

$$\mu < \mu_c = 2\frac{\sqrt{q_1q_2q_3}}{L} + \frac{q_1q_2+q_1q_3+q_2q_3}{L^2} + O(q_i^{5/2})\,. \tag{2.13}$$

For $\mu > \mu_{\rm crit}$, there is a regular horizon at

$$r^2_{+(3)} = \frac{\sqrt{q_1q_2q_3}}{L} + \frac{(q_1q_2q_3)^{3/4}\sqrt{\mu-\mu_c}}{\sqrt{L}}\,. \tag{2.14}$$

In contrast to the previous cases, as the black hole approaches criticality, it maintains a finite sized outer horizon.

The semiclassical entropy and temperature of the black holes under consideration is

$$\begin{aligned} S &= \frac{A}{4G_5} = \frac{2\pi^2}{4G_5}\sqrt{\prod_{i=1}^{3}(r_+^2+q_i)} \\ \beta &= \frac{1}{T} = L^2\frac{2\pi r_+^2\sqrt{\prod_{i=1}^3(r_+^2+q_i)}}{2r_+^6+r_+^4\left(L^2+\sum q_i\right)-\prod_{i=3}^3 q_i}\,. \end{aligned} \tag{2.15}$$

We now apply the above formulas to the cases discussed above:

- $q_2 = q_3 = 0$. In this case,

$$\begin{aligned} S_{(1)} &= \frac{2\pi^2}{4G_5}r^2_{+(1)}\sqrt{q_1} \approx \frac{2\pi^2}{4G_5}\mu\frac{\sqrt{q_1}}{1+q_1/L^2} \\ \beta_{(1)} &\approx 2\pi\frac{\sqrt{q_1}}{1+q_1/L^2}\,, \end{aligned} \tag{2.16}$$

where the approximate equalities are accurate to leading order in small μ.

- $q_3 = 0$. Assuming also that q_2 and q_3 are much less than L^2, one obtains

$$\begin{aligned} S_{(2)} &\approx \frac{2\pi^2}{4G_5}r_{+(2)}\sqrt{q_1q_2} \approx \frac{2\pi^2}{4G_5}\sqrt{(\mu-\mu_{\rm crit})}\sqrt{q_1q_2} \\ \beta_{(2)} &\approx \frac{2\pi\sqrt{q_1q_2}}{r_{+(2)}} \approx \frac{2\pi\sqrt{q_1q_2}}{\sqrt{(\mu-\mu_{\rm crit})}}, \end{aligned} \tag{2.17}$$

where in each line the second approximate equality comes from expanding to non-trivial leading order in small $\mu - \mu_{\rm crit}$.

- All charges non-zero, and much less than L^2. In this case, the entropy is given by:

$$S_{(3)} \approx \frac{2\pi^2}{4G_5}\sqrt{q_1 q_2 q_3} \tag{2.18}$$

At criticality, the temperature vanishes. As μ approaches μ_{crit}, β becomes:

$$\beta_{(3)} \approx \frac{\pi}{L}\sqrt{\frac{q_1 q_2 q_3}{\mu - \mu_{crit}}}\,. \tag{2.19}$$

3. CFT dictionary entries

We now recast the parameters of the above black holes in terms of dual CFT dictionary entries. After also establishing the dictionary entries for the parameters μ and μ_{crit} we state dual CFT formulas for the entropy and temperature.

Although small in comparison to AdS_5, black holes which can be probed by supergravity are still macroscopic objects and as such the CFT operators which describe the black hole will have large scaling dimension $\Delta = ML$. Using the black hole mass formula of equation (2.5) thus yields:

$$\Delta = \frac{\pi L}{4G_5}\left(\frac{3}{2}\mu + q_1 + q_2 + q_3\right). \tag{3.1}$$

In the limit $\mu = q_2 = q_3 = 0$ the CFT dual of the singularity becomes a BPS object with scaling dimension:

$$\Delta_{\text{BPS}(1)} = \frac{\pi L}{4G_5} q_1\,. \tag{3.2}$$

BPS operators in the dual CFT with corresponding $\mathrm{U}(1) \subset \mathrm{SU}(4)_R$ charge J_1 must satisfy the single charge BPS bound:

$$\Delta_{\text{BPS}(1)} - J_1 = 0\,. \tag{3.3}$$

Equation (3.2) then implies:

$$J_1 = \frac{\pi L}{4G_5} q_1\,. \tag{3.4}$$

Using the AdS/CFT dictionary entry $2G_5 N_c^2 = \pi L^3$ we find

$$\frac{J_1}{N_c^2} = \frac{q_1}{2L^2}\,. \tag{3.5}$$

Extrapolating to the other R-charges thus yields

$$\frac{J_i}{N_c^2} = \frac{q_i}{2L^2} \tag{3.6}$$

for $i = 1, 2, 3$. These relations fully determine the corresponding dictionary entry for μ. Equations (3.1) and (3.6) imply

$$\Delta - (J_1 + J_2 + J_3) = \frac{\pi L}{4G_5}\frac{3}{2}\mu \tag{3.7}$$

or,

$$\frac{4}{3}\frac{(\Delta-(J_1+J_2+J_3))}{N_c^2}=\frac{\mu}{L^2}\,. \tag{3.8}$$

The critical operator dimension $\Delta_{\rm crit}$ necessary to form a horizon is thus related to $\mu_{\rm crit}$ by:

$$\frac{4}{3}\frac{(\Delta_{\rm crit}-(J_1+J_2+J_3))}{N_c^2}=\frac{\mu_{\rm crit}}{L^2} \tag{3.9}$$

so that

$$\frac{4}{3}\frac{(\Delta-\Delta_{\rm crit})}{N_c^2}=\frac{\mu-\mu_{\rm crit}}{L^2} \tag{3.10}$$

where

$$\frac{\mu_{\rm crit}}{L^2}=2\left(\frac{(2J_1)\,(2J_2)\,(2J_3)}{N_c^6}\right)^{1/2}+\frac{(2J_1)\,(2J_2)+(2J_1)\,(2J_3)+(2J_2)\,(2J_3)}{N_c^4}+O\left(\frac{J_i^{5/2}}{N_c^5}\right). \tag{3.11}$$

Solving equation (3.9) for $\Delta_{\rm crit}$ yields

$$\Delta_{\rm crit}=\Upsilon+\Delta_{\rm BPS}\,, \tag{3.12}$$

where by abuse of notation, we have defined $\Delta_{\rm BPS}\equiv J_1+J_2+J_3$ and:

$$\Upsilon=\frac{3}{4}\left(2\frac{\sqrt{(2J_1)\,(2J_2)\,(2J_3)}}{N_c}+\frac{(2J_1)\,(2J_2)+(2J_1)\,(2J_3)+(2J_2)\,(2J_3)}{N_c^2}+O\left(\frac{J_i^{5/2}}{N_c^3}\right)\right). \tag{3.13}$$

With the dictionary entries for the q_i's and μ established, we now recast all thermodynamic predictions for the temperature and entropy from supergravity in terms of purely CFT quantities. Making the trivial substitutions, we have for the single, double and triple charge cases:

$$S_{(1)} = \frac{\pi}{N_c}\frac{4}{3}(\Delta-\Delta_{\rm crit})\frac{\sqrt{2J_1}}{1+2J_1/N_c^2} \tag{3.14}$$

$$\beta_{(1)} = \frac{2\pi L}{N_c}\frac{\sqrt{2J_1}}{1+2J_1/N_c^2} \tag{3.15}$$

$$S_{(2)} = \frac{\pi}{N_c}\sqrt{\frac{4}{3}(\Delta-\Delta_{\rm crit})\sqrt{(2J_1)\,(2J_2)}} \tag{3.16}$$

$$\beta_{(2)} = \frac{2\pi L}{N_c}\frac{\sqrt{(2J_1)(2J_2)}}{\sqrt{\frac{4}{3}(\Delta-\Delta_{\rm crit})}} \tag{3.17}$$

$$S_{(3)} = \frac{\pi}{N_c}\sqrt{(2J_1)\,(2J_2)\,(2J_3)} \tag{3.18}$$

$$\beta_{(3)} = \frac{\pi L}{N_c^2}\sqrt{\frac{(2J_1)\,(2J_2)\,(2J_3)}{\frac{4}{3}(\Delta-\Delta_{\rm crit})}}\,. \tag{3.19}$$

Except in the single-charge case, we have assumed that all the J_i are much less than N_c^2; also, we have in all cases expanded to the leading non-trivial order in $\Delta-\Delta_{\rm crit}$. Although we have expressed the entropy of the three possible charge configurations solely in terms of CFT quantities, the CFT temperature predictions still depend on L, which appears to be a purely AdS quantity. Viewing the four dimensional gauge theory as a field theory on the boundary of AdS, the factor of L sets the energy scale for the S^1 of the full $S^3 \times S^1$ geometry of the gauge theory.

It is very tempting to write a general formula which interpolates between the various charge configurations near extremality, at least when all $J_i \ll N_c^2$. By inspection of equations (3.14)–(3.19), the formula which accomplishes this is:

$$\begin{aligned} S &= \frac{\pi}{N_c}\sqrt{\prod_i \left(2J_i + \frac{4}{3}\left(\Delta - \Delta_{\text{crit}}\right)\right)} \\ \beta &= \frac{2\pi L}{N_c}\sqrt{\prod_i \left(2J_i + \frac{4}{3}\left(\Delta - \Delta_{\text{crit}}\right)\right)}\,\frac{\left(1 + \frac{J_1 J_2 J_3}{2N_c}\sqrt{\frac{4}{3}\left(\Delta - \Delta_{\text{crit}}\right)}\right)}{\left(1 + J_1 J_2 J_3\right)\frac{4}{3}\left(\Delta - \Delta_{\text{crit}}\right)}. \end{aligned} \tag{3.20}$$

As we shall see in the next section, so long as it does not vanish, the $J_1 J_2 J_3/N_c$ term in the equation for β is of order at least N_c, justifying the use of the above interpolation formula.

4. Bounds on the semiclassical regime

We now determine the regime of validity for the semiclassical formulas found above. The requirement that the near horizon curvature remain small will also fix the relative sizes of the CFT charges J_i and the large number N_c. One particularly useful measure of the near horizon curvature is:

$$G_5^{4/3} C \equiv \left| G_5^{4/3} R_{\mu\nu} R^{\mu\nu} \right|_{r=r_+} \ll 1. \tag{4.1}$$

The leading order behavior of $R_{\mu\nu}R^{\mu\nu}$ in the various charge cases is given by:

$$\begin{aligned} C_1 &= \frac{1}{L^4}\frac{16}{3}\frac{q_1/L^2}{\left(\mu/L^2\right)^2} + \cdots \\ C_2 &= \frac{1}{L^4}\left(\frac{16}{3}\frac{\left(q_1+q_2\right)/L^2}{\left(\left(\mu-\mu_{\text{crit}}\right)/L^2\right)^2} + \frac{88}{9}\frac{\left(q_1/L^2\right)\left(q_2/L^2\right)}{\left(\left(\mu-\mu_{\text{crit}}\right)/L^2\right)^4}\right) + \cdots \\ C_3 &= \frac{44}{L^4}\left(\frac{q_1}{L^2}\frac{q_2}{L^2}\frac{q_3}{L^2}\right)^{-2/3} + \cdots . \end{aligned} \tag{4.2}$$

Using the dictionary entries established in the previous section, the requirement that the near horizon curvature remain small yields the following constraints on the parameters of the semiclassical regime in the single, double and triple charge cases respectively:

$$N_c^{2/3} \gg \left(\frac{J_1}{\left(\Delta - \Delta_{\text{crit}}\right)^2}\right) \tag{4.3}$$

$$N_c^{2/3} \gg \left(\frac{\left(J_1 + J_2\right)}{\left(\Delta - \Delta_{\text{crit}}\right)^2} + \frac{J_1 J_2 N_c^2}{\left(\Delta - \Delta_{\text{crit}}\right)^4}\right) \tag{4.4}$$

$$N_c^2 \ll J_1 J_2 J_3\,. \tag{4.5}$$

Where in the above bounds we have dropped unimportant factors of order unity. Whereas the single and double charges cases are bounded above by an appropriate power of N_c, in the triple charge case we instead find a lower bound.

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5. Effective strings from giant gravitons

We now present evidence that an effective string created by a distribution of giant gravitons on the spherical factor of $AdS_5 \times S^5$ has the same thermodynamics as a near-extremal R-charged black hole with charges satisfying $q_i/L^2 \ll 1$. (By near-extremal we mean having energies only slightly above the threshold to produce a regular horizon). After analyzing the single charge case in section 5.1, we consider in sections 5.2 and 5.3 more general giant graviton distributions on the S^5 and show that the thermodynamics of the effective strings produced in these cases also match well to the two and three charge black holes.

When the extremality parameter $\mu = 0$, the black hole becomes an uncloaked singularity or "superstar" [12]. Recent work has shown that single charge superstars in AdS_5 are well-described by a distribution of gravitons smeared out along one of the equators of the S^5 [12]. Following [12], we parametrize the S^5 of radius L by angular coordinates $\theta_1, \theta_2, \phi_1, \phi_2$ and ϕ_3 where the three distinct equatorial directions are specified by the three ϕ angles. Assuming that the giant gravitons orbit the S^5 along the ϕ_1 equator, their distribution $dn_1/d\theta_1$ in the θ_1 direction is determined by integrating over the angular dependence of the five-form flux in the other directions of the S^5. This distribution is computed in [12] with the result:

$$\frac{dn_1}{d\theta_1} = N_c \frac{q_1}{L^2} \sin 2\theta_1 \, . \tag{5.1}$$

The total number of such giant gravitons is obtained by integrating over all values of θ_1 for which the five-form flux is positive:

$$n_1 = \int_0^{\pi/2} \frac{dn_1}{d\theta_1} d\theta_1 = N_c \frac{q_1}{L^2} \, . \tag{5.2}$$

As determined in [19], the angular momentum of a single giant graviton orbiting in the ϕ_1 direction and at azimuthal angle θ_1 is:

$$p_{\phi_1} = N_c \sin^2 \theta_1 \tag{5.3}$$

which implies that the total angular momentum of the giant graviton distribution is [12]:

$$P_{\phi_1} = \int_0^{\pi/2} \frac{dn_1}{d\theta_1} N_c \sin^2 \theta_1 d\theta_1 = N_c^2 \frac{q_1}{2L^2} \, . \tag{5.4}$$

Using the dictionary entries previously established, we thus find that the number of giant gravitons is exactly:

$$n_1 = \frac{2J_1}{N_c} \tag{5.5}$$

with net angular momentum:

$$P_{\phi_1} = J_1 \, . \tag{5.6}$$

Based on the "stringy exclusion principle" found in [18], it has been shown that a single giant graviton can have maximal angular momentum N_c before it inflates to the full size of the S^5 [19]. Hence, in addition to the small black hole constraint $J_1 \ll N_c^2$, we must also have $J_1 \gtrsim N_c$ in order to have a finite number of giant gravitons.

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5.1 One distribution of giant gravitons

In this section we present evidence that the transverse intersection of a single D3-brane with the superstar distribution of giant gravitons described above creates an effective string which explains the thermodynamic properties of the single charge black hole. To remain in the regime well-described by both supergravity and the giant graviton distribution, the single charge supergravity bound of equation (4.3),

$$\left(\frac{J_1}{(\Delta - J_1)^2}\right) \ll N_c^{2/3} \tag{5.7}$$

implies:

$$J_1 + \frac{\sqrt{J_1}}{N_c^{1/3}} \ll \Delta \,, \tag{5.8}$$

where we have used the fact that in the single charge case $\Delta_{\text{crit}} = J_1$. Since J_1 scales as $N_c^{1+\delta}$ for some $0 < \delta < 1$, J_1 dominates the lower bound of the above inequality. Thus, although Δ will necessarily be above the BPS bound, it can still scale as $J_1 N_c^{\varepsilon}$ for small $\varepsilon > 0$.

The value of the central charge of the effective string created from the intersection of $2J_1/N_c$ giant gravitons and a single transverse D3-brane follows from the appendix:

$$c_{\text{eff}(1)} = 12\frac{J_1}{N_c} \,. \tag{5.9}$$

Matching the Hagedorn temperature of the effective string to the Hawking temperature of the black hole yields:

$$\frac{L^2}{N_c} = \alpha'_{\text{eff}} \tag{5.10}$$

so that the tension of the effective string becomes:

$$\tau_{\text{eff}} = \frac{1}{2\pi\alpha'_{\text{eff}}} = \frac{N_c}{2\pi L^2} \,. \tag{5.11}$$

Treating the effective string as a real curve inside the giant graviton S^3, the corresponding volume of a real codimension two S^2 is given by:

$$\tau_{\text{D3}} Vol(S^2) = \tau_{\text{eff}} \tag{5.12}$$

so that:

$$Vol(S^2) = \pi L^2 \tag{5.13}$$

which is exactly a quarter the size of the maximum possible S^2 of volume $4\pi L^2$. Hence, the radius of the S^2 never exceeds the maximal value of L. This gives provisional evidence that the intersecting giant graviton picture considered here gives a proper description of single charge black holes in AdS_5: such a black hole slightly above extremality corresponds to an ensemble of highly excited effective strings at their Hagedorn temperature.

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5.2 Two distributions of giant gravitons

As we observed around (2.12), the two-charge black hole in AdS_5 acquires a regular horizon finitely far from the supersymmetric limit: the non-extremality parameter μ must be greater than or equal to $\mu_{\rm crit} \equiv q_1 q_2/L^2$. (Recall that we require $q_i \ll L^2$). This makes it less clear that string theory should provide a correct accounting of the microstates of a black hole with μ slightly larger than $\mu_{\rm crit}$. Nevertheless, we will argue here that an effective string picture once again succeeds in reproducing the thermodynamics up to an overall factor of order unity which we have been unable to determine.

In contrast to the one-charge case where the effective strings are highly excited and the temperature of the black hole close to extremality is identified with the Hagedorn temperature of the strings, here the picture is that an effective string created by the intersection of two distributions of giant gravitons is stretched over a length $L_{\rm eff}$ and has excitations running along it which can be described in terms of a conformal field theory. If this conformal field theory has central charge $c_{\rm eff}$, then up to factors of order unity the free energy of the excitations we have described is

$$F = -c_{\rm eff} L_{\rm eff} T^2 \,, \tag{5.14}$$

where T is the temperature. The entropy, which we assume is carried entirely by the excitations of the conformal field theory on the effective string, satisfies

$$\frac{S}{T} = -\frac{\partial F/\partial T}{T} = 2 c_{\rm eff} L_{\rm eff} \,, \tag{5.15}$$

where we have again neglected factors of order unity. Let us use the supergravity results (3.16) and (3.17) for S and T in the two-charge case to estimate

$$2 c_{\rm eff} L_{\rm eff} = 8\pi^2 L \frac{J_1}{N_c}\frac{J_2}{N_c} \,. \tag{5.16}$$

The effective string arises from the intersection of two distributions of giant gravitons orbiting different equators of the S^5, with $2J_1/N_c$ giant gravitons along one equator and $2J_2/N_c$ along the other. As determined in the appendix, the central charge of the effective string is

$$c_{\rm eff(2)} = 12 \frac{J_1}{N_c}\frac{J_2}{N_c} \,. \tag{5.17}$$

Combining (5.16) and (5.17), the effective string has length

$$L_{\rm eff(2)} = \frac{\pi}{6}(2\pi L) \,, \tag{5.18}$$

which is a well behaved length for a circle on the S^5 of radius L. As with the final result (5.13) of section 5.1, the main point is that elementary reasoning based on simple string theory pictures give the correct number of microstates to understand the semiclassical black hole entropy.

5.3 The three-charge case in terms of the effective string

The successful description of black hole microstates of the double charge black holes suggests that the three-charge case should not be much different, at least in a limit where the third charge is much smaller than the first two. The reason is that (again in analogy with the asymptotically flat space cases) the third charge can then be regarded as momentum along the string in one direction.

As with the two-charge case, one must proceed finitely above the supersymmetric mass limit to achieve a regular horizon. We will be unable to give any account of this finite mass gap, and we will refer only to entropy and conserved charges. We note however that for $q_3 \ll q_1 q_2/L^2$, the mass gap becomes the same for the two and three-charge cases (compare (2.13) with the discussion above (2.12)). This would thus be a good starting point for understanding why there is some mass gap in the two charge case.

We propose to describe the microstates behind the zero-temperature horizon that forms exactly at $\mu = \mu_{\text{crit}(3)}$ in terms of states of the effective string where there are excitations moving only in one direction. Such states can indeed have finite entropy and zero-temperature. Their entropy comes from Cardy counting:

$$S_{\text{Cardy}} = 2\pi \sqrt{\frac{n c_{\text{eff}}}{6}}\,, \tag{5.19}$$

where n is the level of the excitations moving in one direction: thus $n = J_3$. According to the discussion above and the computation performed in the appendix, we should use the same c_{eff} as in the two-charge case:

$$c_{\text{eff}(3)} = c_{\text{eff}(2)} = 12 \frac{J_1}{N_c} \frac{J_2}{N_c}\,. \tag{5.20}$$

Comparing (5.19) to the result (3.18), one sees that the level of the effective string would need to be:

$$n = J_3\,. \tag{5.21}$$

This agrees with our previous claim that n should be identified with J_3.

Just as the individual graviton could have maximal angular momentum N_c, for an effective string picture to be accurate, the highest mode excitation n must be bounded above by N_c. To describe the case where all three q_i are comparable, one would need to consider the quantum mechanics of three distributions of giant gravitons. This appears to be a formidable problem.

In summary, the account of microstates based on intersecting giant gravitons and effective strings agrees quite well with the entropy of the corresponding charged black holes in AdS_5. The main puzzle remaining is the mass gap in the two and three charge cases between the supersymmetric bound on the mass and the minimum mass needed to form a regular horizon. Qualitatively, what has to happen in the CFT is that there are very few operators with $\Delta - J_1 - J_2 - J_3$ less than this mass gap, but for larger "twist" there are such a profusion of operators that they correspond to macroscopic horizons in AdS_5.

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6. The one-charge black hole and free fermions

In light of recent results [20, 21] on the description of supersymmetric states asymptotic to $AdS_5 \times S^5$ with one non-zero angular momentum, it is interesting to inquire how the free fermion picture might elucidate the physics of the single charge case. For $\mu = 0$ and only $q_1 \neq 0$, the solution (2.1) with $k = 1$, when lifted to ten dimensions, can be recast in the form described by [21, eq. (2.5)].[1]

The metric in ten dimensions [12] is

$$\begin{aligned} ds^2 &= \sqrt{\Delta}\left[-\frac{f}{H}dt^2 + \frac{dr^2}{f} + r^2 d\Omega_3^2\right] + \\ &+ \frac{1}{\sqrt{\Delta}} H\left[L^2 d\mu_1^2 + \mu_1^2\left(L d\phi_1 + \left(\frac{1}{H} - 1\right) dt\right)^2\right] + \\ &+ \frac{1}{\sqrt{\Delta}}\sum_{i=2}^{3} L^2(d\mu_i^2 + \mu_i^2 d\phi_i^2)\,, \end{aligned} \tag{6.1}$$

where the μ_i are constrained by $\sum_{i=1}^3 \mu_i^2 = 1$, the functions f and H are as given in (2.3) and (2.4), and $\Delta = H - \mu_1^2(H-1)$. The second 3-sphere, $\tilde{S}^3$ in the notation of [21], is in the hyperplane parametrized as $(\mu_2\cos\phi_2, \mu_2\sin\phi_2, \mu_3\cos\phi_3, \mu_3\sin\phi_3)$, and its radius (from the metric (6.1)) is $L\sqrt{1-\mu_1^2}/\Delta^{1/4}$. The transformation of variables needed to bring (6.1) into the form described by of [21, eq. (2.5)] includes

$$\begin{aligned} \sqrt{x_1^2 + x_2^2} &= \mu_1\sqrt{r^2 + L^2 + q_1} \\ y &= rL\sqrt{1-\mu_1^2}\,, \end{aligned} \tag{6.2}$$

and the all-important function z of [21] is

$$z = \frac{1}{2}\frac{r^2 + (q_1 - L^2)(1-\mu_1^2)}{r^2 + (q_1 + L^2)(1-\mu_1^2)}\,. \tag{6.3}$$

It is possible to use (6.2) to eliminate μ_1 and r and express z solely in terms of y and $\sqrt{x_1^2 + x_2^2}$. The resulting expression is somewhat lengthy, and we will not record it here. Its $y \to 0$ limit is quite simple, though:

$$z\Big|_{y=0} = \begin{cases} \dfrac{1}{2}\dfrac{q_1 - L^2}{q_1 + L^2} & \text{if } x_1^2 + x_2^2 < q_1 + L^2 \\ \dfrac{1}{2} & \text{otherwise.} \end{cases} \tag{6.4}$$

If $q_1 = 0$, then we recover the free fermion description of $AdS_5 \times S^5$, which is N_c fermions filling a disk-shaped droplet of radius L in the x_1-x_2 plane. The one-charge superstar is some excitation of this state, and from (6.4) we see that it is a uniformly but partially filled disk of radius $\sqrt{q_1 + L^2}$. The filling fraction must be $\nu = L^2/(q_1 + L^2)$ in order for

[1] The explicit form of the solution that we present here is known to O. Lunin and J. Maldacena [22].

the number of fermions still to be N_c. Comparing with (6.4), we arrive at the result

$$z\Big|_{y=0} = \frac{1}{2} - \nu \,. \tag{6.5}$$

This partially filled state is not unique: it should be regarded as an ensemble where the quantum states of individual fermions corresponding to points inside the disk each have probability ν of being occupied. Any representative of this ensemble may be constructed from the ground state by acting with a color-singlet polynomial in the complex scalar field $Z_1 = X_1 + iX_2$ — the particular polynomial being constructed from factors corresponding to Young diagrams in a manner that is described quite precisely in [20, 21]. A rough approximation of this construction is to think of many powers of $\det Z$ and sub-determinants of it multiplied together to give the operator used for constructing a representative state of the ensemble. These products are the CFT translation of the distribution of giant gravitons found in [12].

It is interesting to write the thermodynamics of the near-extremal single-charge superstar in terms of ν: we recall from (3.14) that

$$S = \frac{4\pi}{3}(\Delta - J_1)\frac{\sqrt{2J_1/N_c^2}}{1 + 2J_1/N_c^2} = \frac{4\pi}{3}(\Delta - J_1)\sqrt{\nu(1-\nu)} \,, \tag{6.6}$$

in a regime where $1 \ll \Delta - J_1 \ll J_1$. This is quite an interesting regime because as one increases the "twist"" $\Delta - J_1$ from 0, the singular BPS solution gradually grows a regular horizon. Counting the number of states with large J_1 and moderate $\Delta - J_1$ seems closer to tractable than other entropy problems encountered in the study of strongly coupled $\mathcal{N} = 4$ super Yang-Mills. Perhaps surprisingly, the expression for the entropy in (6.6) involves no explicit factors of N_c, encouraging us to think that a correct counting of color singlets will provide a complete enumeration of the relevant microstates.

7. Conclusions and outlook

We have shown that the salient thermodynamic features of R-charged black holes in AdS_5 can be understood in terms of effective strings created from the intersection of giant gravitons. In the single charge case the free fermion picture of [21] make it possible to give a very explicit description of the dual CFT operators, and we have briefly indicated in section 6 how they can be constructed. In this concluding section we consider the two and three charge cases. Letting Z_i denote the three complex scalars charged under $UU(1)^3 \subset \mathrm{SU}(4)_R$, the operators $\det Z_i$ each correspond to a maximal giant graviton with angular momentum along a distinct equator of the S^5 [23].

Upon identifying the two distributions of giant gravitons with operators built from Z_1 and Z_2 fields, the effective string moves along the third equator of the S^5 and thus consists of N_L left-moving Z_3 fields and N_R right-moving $\bar{Z}_3$ fields in the dual CFT. The total dimension of a corresponding gauge theory operator is then:

$$\Delta = N_L + N_R + J_1 + J_2 + \delta_{imp} \tag{7.1}$$

where the presence of the additional small impurity term δ_{imp} follows from the fact that the operators must remain above the BPS bound.[2] As of the writing of this paper, we do not yet know of a systematic way to insert the Z_3 and $\bar{Z}_3$ fields and the order δ_{imp} of other impurities and will thus simply write the microstates of such black holes, $\{\Omega_{\rm BH}\}$ as:

$$\{\Omega_{\rm BH}\} \sim \left\{ \prod_{i=1}^{J_1/N_c} \int \det X(x_i)\rho\,(x_i)\, d^4x_i \prod_{j=1}^{J_2/N_c} \int \det Y(y_j)\rho\,(y_j)\, d^4y_j \right\}_{\substack{\text{all possible insertions of} \\ N_L\ Z's,\ N_R\ \bar{Z}'s \\ \delta_{\rm imp}\ \text{other impurities}}} \tag{7.2}$$

where the presence of the $d = 4$ distribution ρ simply indicates that we do not yet understand how localized the operators are on the CFT and the product over determinants runs from 1 to J_1/N_c rather than $2J_1/N_c$ because the determinant prescription we are using is still too crude to correctly model angular distributions of giant gravitons on the S^5. It would be interesting to explicitly verify that the entropy generated by all such gauge theory operators correctly matches onto the known black hole entropy. Finally, we recall that in contrast to the single charge case where a black hole can form for an arbitrarily small value of $\mu > 0$, in both the double and triple charge black holes, the extremality parameter must remain above the finite value $\mu_{\rm crit} > 0$. This behavior suggests the possibility of a phase transition in passing from the single to higher charge cases. This should correspond to a sufficiently large value of δ_{imp} in the dual CFT, but we do not yet understand the dynamics responsible for this phase transition. Perhaps it is analogous to a free energy of crystallization, where the microstates of the black hole are like the high-entropy, high-energy fluid phase and the BPS state is like the crystalline phase.

As the above analysis demonstrates, even the smallest semiclassical R-charged black holes must have R-charge at least N_c in order for the intersecting giant graviton picture to produce a physically reasonable effective string. Although the effective strings for black holes in flat space and AdS_5 are both produced from the intersection of D3-branes, there are some important differences between the properties of these strings. Indeed, whereas black holes in flat space are described by effective strings produced by order J D3-branes, black holes in AdS_5 are described by effective strings produced by only order J/N_c D3-branes. The ubiquitous factors of N_c in both the supergravity formulas and effective string relations underscore the key role that the five-form flux plays in the dynamics of extended objects in $AdS_5 \times S^5$. In any case, a more detailed description of the dual CFT operators for such configurations would be most instructive.

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[2] See equation (3.12).

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A. Intersection numbers and the central charge

In this appendix we compute the central charge of the effective string in the two and three-charge cases by calculating the intersection number of two giant graviton distributions sourced by charges q_1 and q_2. The case of one giant graviton distribution intersecting a single transverse D3-brane follows as a special case of the following analysis and will be omitted. Using notation established in section 5, we introduce the directional cosines:

$$\mu_1 = \cos\theta_1,\ \mu_2 = \sin\theta_1\cos\theta_2,\ \mu_3 = \sin\theta_1\sin\theta_2\,. \tag{A.1}$$

The self-dual RR five-form flux is then given by:

$$F^{(5)} = dB^{(4)} + *dB^{(4)}\,, \tag{A.2}$$

where [12]:

$$B^{(4)} = -\frac{r^4}{L}\Delta dt\wedge d^3\Omega - L\sum_{i=1}^{3} q_i\mu_i^2\left(Ld\phi_i - dt\right)\wedge d^3\Omega \tag{A.3}$$

with $d^3\Omega$ the volume element of a unit S^3 with angles $(\alpha_1,\alpha_2,\alpha_3)$. Since Δ only depends on the radial coordinate r, we will not need its explicit form in the computation to follow. Following [12], the first distribution of giant gravitons sourced by charge q_1 generates a flux $F^{(5,1)}$ with component along the S^5:

$$F^{(5,1)}_{\theta_1\phi_1\alpha_1\alpha_2\alpha_3} = L^2 q_1 \sin 2\theta_1 \sin^2\alpha_1\sin\alpha_2\,. \tag{A.4}$$

Integrating over all coordinates except θ_1 yields the number distribution of gravitons in the θ_1 direction:

$$dn_1 = \frac{N_c}{4\pi^3 L^4}d\theta_1\int F^{(5,1)}_{\theta_1\phi_1\alpha_1\alpha_2\alpha_3}d\phi_1 d^3\alpha = \frac{2J_1}{N_c}\sin 2\theta_1 d\theta_1\,, \tag{A.5}$$

where in the last equality we have used the dictionary entries previously established. Similarly, the second distribution of giant gravitons sourced by charge q_2 generates two components along the S^5:

$$\begin{aligned} F^{(5,2)}_{\theta_1\phi_2\alpha_1\alpha_2\alpha_3} &= -L^2 q_1 \sin 2\theta_1\cos^2\theta_2\sin^2\alpha_1\sin\alpha_2 \\ F^{(5,2)}_{\theta_2\phi_2\alpha_1\alpha_2\alpha_3} &= L^2 q_1 \sin^2\theta_1 \sin 2\theta_2\sin^2\alpha_1\sin\alpha_2\,. \end{aligned} \tag{A.6}$$

The number distribution of giant gravitons due to the q_2 charge is then:

$$dn_2 = \frac{2J_2}{N_c}\left(\sin^2\theta_1\sin 2\theta_2 d\theta_2 - \sin 2\theta_1\cos^2\theta_2 d\theta_1\right)\,. \tag{A.7}$$

To determine the intersection number of these two distributions, we consider the intersection of the infinitesimal distributions at angles θ_1 and θ_2 so that the spheres intersect over some S^1. Now perform the Hopf fibration along this S^1:

$$\varphi : S^5 \to S^5/S^1 \simeq \mathbb{CP}^2\,. \tag{A.8}$$

Since an S^3 inside of the S^5 is given by a degree 2 real algebraic variety and since the giant graviton distribution at this angle is this same S^3 wrapped dn times, the first distribution of giant gravitons maps to a complex curve $C_1(\theta_1)$ in $\mathbb{CP}^2$ of degree $2dn_1$ and the second distribution of giant gravitons maps to a complex curve $C_2(\theta_2)$ of degree $2dn_2$. The total intersection number $I(\theta_1,\theta_2)$ of the curves $C_1(\theta_1)$ and $C_2(\theta_2)$ is then given by Bézout's theorem, which states that $I = \deg C_1 \deg C_2$, or:

$$I(\theta_1,\theta_2) = 4dn_1 dn_2\,. \tag{A.9}$$

To determine the total intersection number of the two distributions, we recall that we are simply counting the number of units of flux from the five-form flux generated by the giant gravitons. For this reason, we must integrate the number distributions of equations (A.5) and (A.7) over their positive domains of definition, $\theta_1,\theta_2 \in [0,\pi/2]$. The total intersection number is thus:

$$\begin{aligned} I_{\text{tot}} &= \int_0^{\pi/2}\int_0^{\pi/2} I(\theta_1,\theta_2)d\theta_1 d\theta_2 & \text{(A.10)}\\ &= 16\frac{J_1}{N_c}\frac{J_2}{N_c}\int_0^{\pi/2}\sin^2\theta_1\sin 2\theta_1 d\theta_1\int_0^{\pi/2}\sin 2\theta_2 d\theta_2 = 8\frac{J_1}{N_c}\frac{J_2}{N_c}\,. & \text{(A.11)}\end{aligned}$$

This I_{tot} corresponds to the bosonic central charge of the string. Assuming that the effective string has some supersymmetry, each worldsheet boson has a fermionic counterpart so that the total central charge for the effective string is:

$$c_{\text{eff}} = \frac{3}{2}I_{\text{tot}} = 12\frac{J_1}{N_c}\frac{J_2}{N_c}\,. \tag{A.12}$$

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